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Analysis of Voting procedures in One-Seat Elections: Condorcet Efficiency and Borda Efficiency

by

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DISCUSSION
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Analysis of voting procedures in one-seat elections: Condorcet efficiency and Borda efficiency

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Abstract

In this paper 16 different voting procedures for one-seat elections are analysed: the rules of Borda, Condorcet, Black, Copeland, Simpson, Hare, Coombs, Baldwin, Nanson and the plurality, anti-plurality, majority, approval and runoff rules. The 2 criteria we use as a measure for the validity of the voting procedures are Condorcet efficiency (the number of times a voting procedure selects the Condorcet winner) and Borda efficiency (the number of times a voting procedure selects the Borda winner). Computer simulations calculate efficiencies for the 16 voting procedures. We find that the Borda rule is about 85% Condorcet efficient while some voting procedures are always 100 % Condorcet efficient (Black, Copeland, Simpson, Baldwin, Nanson). Another rule is only 100 % Condorcet efficient with single peaked profiles (Coombs). This can be proven theoretically. Another feature from single peaked profiles seems to be that some voting procedures select the same winner (Simpson, Baldwin, and Nanson). This result may be interesting for future research. Considering Borda efficiencies we see that the Black rule scores well, followed by the rules of Copeland, Nanson and Baldwin. Taking both Condorcet and Borda efficiencies into account, we can state that the Black rule is superior. Then come the rules of Copeland, Simpson, Nanson and Baldwin.

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1. Introduction

Since Arrow's Impossibility Theorem, it is widely known that no voting procedure is ideal in terms of Arrow's conditions. Therefore a second best procedure should be constructed, albeit somewhat difficult to determine on which basis we should select such a second best procedure. There are multiple desirable criteria to choose from. In this paper, we concentrate on two special criteria: Condorcet efficiency and Borda efficiency, i.e. the number of times a voting procedure selects the Condorcet winner, if there exists one, and the number of times the Borda winner is elected. In essence, we consider the Borda and Condorcet rules as valuable. An elaborated motivation for this opinion will be given below.

In section 2, we develop the theoretical framework of present research. Essentially, we assume that voters are able to give a full ranking of all available alternatives and that the voting procedure is aimed at the selection of one winner. In a third section we develop a computer simulation in order to compute both Condorcet and Borda efficiencies of different voting procedures. The results from this simulation are analysed thoroughly. In a concluding section, we focus on the main contributions from this simulation.

2. The set up of the model

2.1 Definitions and assumptions

Throughout the paper we assume there is a finite set of n voters and m alternatives. Trivial cases with $n < 3$ or $m < 2$ are excluded. An abstract formulation of alternatives is given by *a*, *b*, *c*, ... and of voters by *1*, *2*, *3*, ... The preference orderings per voter are supposed to be strict linear orderings and binary relations are irreflexive, transitive and complete. The i-th position on a preference ordering is called rank i ($i = 1, 2, ..., m$). The n-tuple of all voters' preference orderings is called a profile. Anonymity of voters makes it possible to take voters with the same preference ordering together.

A voting procedure is a prescribed system that starting from the full profile calculates the social choice. Different kinds of social choice are possible: no winner (nobody got sufficient votes)[2], exactly one winner, several tied winners or a "*social ordering*". We don't consider the case of social orderings in this paper. We point out that we are only interested in the one and only winner (if one exists) as we want to fill the one "seat" available.

2.2 Different voting procedures

We analyse 3 different types of voting procedures in one-seat elections: point systems, Condorcet-consistent rules and elimination procedures. As we assume the voters to be "time consistent" in their preferences we only look at one-ballot systems, where the voters have to

[2] Voting procedures are not always able to determine a winner, which is described as non-determinist.

reveal their full rankings at once. Furthermore, we also work with the full profile at once, i.e. sequential procedures[3] are not analysed.

2.2.1 Point systems

In order to use a ***point system*** we assign s_i points to an alternative when it has rank i in a voter's preference ordering. In this way each voter assigns the following points (gathered in a points vector) to the alternatives from rank 1 to rank m: $(s_1, s_2, s_3, ...s_m)$ with $s_1 \geq s_2 \geq s_3 ... \geq s_m$ and $s_1 > s_m$. Adding up the points received from all voters gives an alternative's total score. The alternative with the highest score wins.

The point systems we use are the *plurality rule* with points vector (1,0,0,...,0,0), the *Borda count* with points vector (m-1,m-2,m-3,....,1,0), the *anti-plurality*[4] with points vector (1,1,1,...,1,0) and *approval voting* with points vector $(1,s_2,s_3,...,s_{m-1},s_m)$ where s_i (i = 2,3,..,m) is equal to 1 or 0, to be determined by each voter individually, but with $s_j = 0$ if $s_{j-1} = 0$ (j = 3,4,..,m). The *majority rule* selects the plurality winner if she receives a score larger than n/2, which means that more than half of the electorate puts her on rank 1.

2.2.2 Condorcet consistent rules

Condorcet consistent rules start from pairwise comparisons between all alternatives. A so-called Dodgson matrix can be computed in the following way. Define $x_{i,j}$ as the number of voters who place alternative i higher (i.e. closer to rank 1) in their preference ordering than alternative j. Clearly, $x_{i,i} = o$ and $x_{i,j} + x_{j,i} = n$. The m by m Dodgson matrix is defined as $[x_{i,j}]$. The *Condorcet* winner, if one exists, beats all alternatives in pairwise comparisons, hence alternative i is the Condorcet winner if all $x_{i,j} > (n/2)$ with j = 1,2,i-1,i+1,...,m. Note that there is one Condorcet winner at most (the Condorcet rule is not determinist) and that in the case a majority winner exists this is the Condorcet winner. One important feature we have to mention (here without proof) is that the sum over all j's from $x_{i,j}$ gives the Borda score for alternative i. *Black's procedure* is described as: if a Condorcet winner exists, then this is the Black winner, else choose the Borda winner.

Simpson proposed a kind of maximin rule on the Dodgson matrix. For all i, take the smallest $x_{i,j}$ with j = 1,2,i-1,i+1,...,m and call that one x_i. Alternative i with the largest x_i wins. In order to use *Copeland*'s rule we have to write another m by m matrix, defined as $[y_{i,j}]$, in the following way: write 0's on the diagonal, and put (for $i \neq j$) $y_{i,j} = 1$, 0 or -1 if $x_{i,j}$ is larger than, equal to or smaller than n/2 respectively. The sum over all j's from $x_{i,j}$ gives the Copeland score for alternative i. The alternative with the highest Copeland score wins.

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[3] We describe a sequential voting procedure as one that in sequential stages computes a winner out of a certain number of alternatives (but not all of them) and shifts only this winner to a new round *where a number of not yet participating alternatives are brought in*, computes again a winner and so on.

[4] Anti-plurality is also known as inverse plurality.

2.2.3 Elimination procedures

The *two-stage runoff procedure* or shorter *runoff procedure* needs one or two stages to select a winner. If a majority winner exists, then this is the runoff winner and the procedure stops. Otherwise, consider the plurality scores of all alternatives. Withhold the two alternatives with the highest plurality scores and put them in a pairwise comparison. The winner from this round is the runoff winner. We programmed this procedure strictly in the computer simulation, i.e. the procedure stops and declares nobody as a winner in any case of ties.

The *Hare procedure*[5] uses the plurality scores at each stage. Now, the alternative with the lowest plurality score is eliminated from the profile. We get a new profile with the remaining alternatives where the relative position between all these alternatives is exactly the same as in the original profile. Again appropriate plurality scores are calculated and the procedure continues until one winner is found. In the strict version of this rule, the procedure stops as soon as there is a tie and no winner is selected. In a weaker version we can allow for ties at each stage and eliminate all alternatives in the tie when in a non-final stage[6] and declare all alternatives as winners when in the final stage of the procedure.

The *Coombs procedure*[7] uses the anti-plurality scores at each stage. Now, the alternative with the lowest inverse plurality score (the largest number of last rank places) is eliminated from the profile. As with Hare's procedure we get a new profile, again inverse plurality scores are calculated and the procedure continues until one winner is found. Concerning ties, the same strict and weak version of the Coombs procedure is used as we did by the Hare procedure.

The *Nanson procedure* starts from the initial Borda count. All alternatives that fail to achieve an equal or more than average Borda score of all alternatives are eliminated and the Borda count is run amongst the remaining alternatives. Thereby the profile is re-written with the remaining alternatives only, and new Borda points are assigned. If k out of the m alternatives are already eliminated, then the Borda points vector looks like (m - 1 - k, m - 2 -k, ..., 1, 0) with (m - k) elements. The total number of Borda points each voter gives equals the sum of the (m-k) elements from the Borda points vector, i.e. (m-1-k) * (m-k) / 2. With n voters, the total number of Borda points to be shared equals n * (m-1-k) * (m-k) / 2. There are m - k alternatives remaining, hence the average per remaining alternative or "quotum" equals n * (m-1-k) / 2. So, at each stage all alternatives that fail to achieve this quotum are eliminated and the procedure continues. The last surviving alternative is declared as winner. Note that in the case where at any stage where the Borda scores of the remaining alternatives are equal[8],

[5] The Hare rule is also known as plurality runoff, alternative vote or single transferable vote for one-seat elections

[6] We are in non-last stage if not all remaining alternatives are in a tie. Then those tied alternatives are all eliminated and the procedure continues with the surviving alternatives. We are in a last stage if all remaining alternatives are in a tie.

[7] The Coombs rule is also known as inverse plurality runoff or exhaustive voting

[8] Note that there are cases where only one Nanson winner can emerge. Take e.g. n odd and m - k = 2, then the 2 alternatives can't get an equal Borda score.

all of them are declared as tied winners as each alternative gets the quotum and cannot be eliminated. If this does not happen, then the last stage is nothing else than use of the Condorcet criterion between the two remaining alternatives.

The *Baldwin procedure* [9] is a modification of Nanson's method in the sense that only one alternative at a time is eliminated, i.e. this one with the lowest Borda score. The procedure then continues with the remaining alternatives, new Borda points are assigned and so on. The procedures from Baldwin and Nanson need not to select exactly the same winners.

3. Computer simulation

In this section, we simulate elections by computer. Given a randomly chosen profile, the winner(s) of the different voting procedures are calculated. We work with randomly chosen profiles instead of "real" profiles because we are interested in general results. Even with a limited number of simulations, some indications may emerge which can be interesting to further analyse in theory (i.e. find a theoretical proof). Another advantage of simulations is that one can vary the number of voters and alternatives.

The idea to generate elections by computer in this kind of framework comes from the article *A Comparison of Efficiency of Multialternative Electoral Systems* from Samuel Merrill (1984). Merrill tested the Condorcet efficiency of 7 voting procedures. The far-famed table with his results is given below.

Table 1: Condorcet efficiencies for a random profile with 25 voters by Merrill (1984)

procedure \ # alternatives	2	3	4	5	7	10
BORDA	100,0	90,8	87,3	86,2	85,3	84,3
CONDORCET	100,0	100,0	100,0	100,0	100,0	100,0
PLURALITY	100,0	79,1	69,4	62,1	52,0	42,6
APPROVAL	100,0	76,0	69,8	67,1	63,7	61,3
BLACK	100,0	100,0	100,0	100,0	100,0	100,0
RUNOFF	100,0	96,2	90,1	83,6	73,5	61,3
HARE (TIES)	100,0	96,2	92,7	89,1	84,8	77,9
COOMBS (TIES)	100,0	96,3	93,4	90,2	86,1	81,1
% Condorcet winners	100,0	91,6	83,4	75,8	64,3	52,5

Source: Merrill (1984)

Our current computer simulation program is written in Turbo Pascal 7.0. We simulate elections with a number of voters ranging from 25 to 425 and a number of alternatives ranging from 2 to 20. The number of elections simulated (the number of "runs") was each time 10.000.

[9] We use the name "Baldwin" rule in order to stress the difference with the Nanson rule. John Taplin mentioned that name on the Election Methods Internet Site. The procedure may be better known as e.g. Nanson's modification of his own rule (cf. McLean and Urken (1995)), but this terminology is confusing.

In our program both Condorcet and Borda efficiencies are calculated. A justification for the use of both Condorcet and Borda efficiencies will be given here. First of all, Jean-Charles de Borda and Marquis de Condorcet had historical discussions about the validity of their system and this interesting discussion is still going on. Second, it is clear that voting procedures by which the full profile is not taken into account can give winners that are not well-supported by the electorate. But the Condorcet and Borda rule do make use of the full profile, albeit in two different ways. Furthermore, the selection of the Condorcet winner generally can be seen as a strong concept as this alternative beats *all* other alternatives by pairwise comparisons. However, some problems arise by stressing uniquely this feature. First, there is not always a Condorcet winner, which is an important drawback of the Condorcet rule. In general, if there exists a Condorcet winner, then this is mostly a strong alternative. In a small number of cases however, a Condorcet winner needs not always to be the “ideal” social choice, as in the following case:

a	b	b
b	c	d
c	d	c
d	a	a
5	2	2

The Borda winner b seems to enjoy a larger overall support than Condorcet winner a. Note that even the widely accepted majority rule provides the same “undesirable” alternative a as winner. Therefore it is our conviction that it is worthwhile to consider not only the Condorcet rule, but also the Borda rule. The Borda rule *always* gives a winner and due to its points system, the Borda winner must be a “widely accepted” alternative. Naturally, even the Borda rule has drawbacks such as manipulability, but this kind of analysis falls beyond the scope of this paper.

Additionally in the computer program, the concepts strong and weak efficiency are introduced and used for both Condorcet and Borda efficiency[10]. An important expansion in comparison with Merrill’s program is the inclusion of other voting procedures: majority, anti-plurality and the rules by Copeland, Simpson, Nanson and Baldwin. Remark that we include both the strict and the less strict versions of Hare’s and Coombs’ rule.

In a slightly different version of our basic program, we calculate Condorcet and Borda efficiencies on the more restricted domain of single peaked profiles. On the following lines the basic program will be dealt with, in the next section the program for single peaked preferences.

[10] For more details, see below.

3.1 Basic program

3.1.1 Set-up of the basic program

The program contains four major steps. In the first step, we can choose the number of alternatives and the number of voters. First we follow Merrill by choosing 25 voters. In order to control for the difference between an even and an odd number of voters, we also look at cases with 26 voters.

In the second step, for a given number of voters and alternatives the computer calculates the winner(s)[11] of the different voting procedures. All procedures, except approval voting, can be run immediately starting from the given profile, in the same way as described above. For approval voting, we need to use some kind of a trick[12].

Concerning the procedures from Hare and Coombs, two different versions are used. A first strict one does not allow for ties, in the sense that the procedure stops and is not able to declare a winner in the case of ties. A second one does allow for ties. The two-stage runoff procedure is programmed in its strict version i.e. it stops in the case of ties. The Nanson and Baldwin procedures are programmed to allow for ties in the same way as with Hare and Coombs.

The third step of the simulation consists of repeating 10.000 times the above described two steps. Then we get the 10.000 Borda winners, 10.000 plurality winners, and so on. Remark that only in the case of Condorcet, two-stage runoff and the strict versions of Hare and Coombs, there is not always a winner.

The fourth and last step of the program consists of calculating the Condorcet and Borda efficiencies. Condorcet efficiency is only applied to those runs where there is a Condorcet winner.

The difference between strong and weak efficiency is directly related to the difference between a strong and weak intersection between the winner(s) from voting procedure X and voting procedure Y. When both voting procedures select exactly the same winner or winners, then we talk about a strong intersection. In the case there is no strong intersection, but at least one of the winners from the two voting procedures is the same, we have a weak intersection. If we define the set of winners from voting procedure X as X( ) and the set of winners from voting procedure Y as Y( ), e.g. X(A,C) and Y(A,C) are strong intersections and X(A,B,C) and Y(A,B) are weak intersections. Remark that cases like X(A,B) and Y(C) or X(A) and Y(no winner) or X(no winner) and Y(no winner)[13] are not considered to be intersections.

[11] If there does not exist a winner, then the position of the winner(s) is filled with a blank.

[12] We constructed it as follows: a voter gives different numbers to the alternatives, all the alternatives with a number above the average are given 1 point, the others get 0.

[13] We want to measure the number of times a winner is selected by another voting procedure. Therefore we cannot take X(no winner) and Y(no winner) into account.

There can only be a weak intersection if at least one of the two voting procedures gives tied winners.

Strong (weak) Condorcet efficiency of voting procedure X can be defined as the number of times there is a strong (weak) intersection between the Condorcet winner and the winner(s) of voting procedure X, divided by the number of times a Condorcet winner exists.

Strong (weak) Borda efficiency of voting procedure X can be defined as the number of times there is a strong (weak) intersection between the Borda winner(s) and the winner(s) of voting procedure X, divided by the number of runs (in our case this is 10.000).

3.1.2 Results from the simulations

All tables with results are in the appendix. First we concentrate on the *strong Condorcet efficiency* results, given in table A.1. These are the results from 10.000 elections with *25 voters*. This process was repeated 19 times for a number of alternatives ranging from 2 to 20.

On the last line of the table, the percentage of Condorcet winners is given. For 2 alternatives there should always be a Condorcet winner as the number of voters is odd[14]. As the number of alternatives increases, the probability to find a Condorcet winner decreases[15]. With m alternatives the Condorcet winner has to beat the other m - 1 alternatives, and this is more difficult with m increasing. It follows that with an increasing number of alternatives Condorcet efficiency is calculated for a decreasing number of runs.

By looking at the table, three findings appear. First, for 2 alternatives, there is always 100 % efficiency, which is a trivial result. Second, there are 6 voting procedures that always select the Condorcet winner. Third, the Condorcet efficiency of the other voting procedures declines with an increasing number of alternatives. We elaborate these 2 latter findings in the next paragraphs.

The Condorcet rule clearly is and the rules from Black, Copeland and Simpson are it by definition. Nanson did already see that there is a direct relationship between his rule and the Condorcet rule. We don’t use his proof; instead we give an easier proof in the appendix.

Concerning the other voting procedures, the results are self-evident. The Borda count selects in more than 80 % of the cases the Condorcet winner. The other procedures select the Condorcet winner in a decreasing way as the number of alternatives increases. Let us e.g. concentrate on the elimination procedures. The weak performance of the strict Hare and Coombs rules are due to the large number of cases where the procedures stops because of ties. For 3 alternatives they perform rather well, but with more alternatives, more ties do

[14]Remark that this 100 % Condorcet winners result will be lower than 100% if the number of voters is even because then ties do occur. See below.

[15]Our calculated percentages are similar to the ones Fishburn got in his “Probability of no Condorcet winner” table, cited in Moulin (1988).

occur with no declaration of a winner as result. If we allow for ties, then they perform much better. Especially the Coombs rule is good at selecting the Condorcet winner[16]. The Hare rule is more likely to eliminate the Condorcet winner than the Coombs rule. The intuition behind this result is the following: if alternative i is the Condorcet winner, then we see that alternative i almost never appears on the last or almost last ranks as it has to beat all the other alternatives by pairwise comparison. By using the Coombs rule, the probability to eliminate this alternative is small. One could tell a similar story for the Hare rule. However we see that the Condorcet winner need not at all to appear on the first rank. So there is a slightly larger probability that the Condorcet winner loses from an other alternative by taking the first rank into account than that she loses from an other alternative by taking the last rank into account.

Approval voting is performing rather well compared with the plurality rule. The runoff procedure performs in a similar "bad" way as the plurality rule. This must be due to the specific set-up of this procedure, which is based on the plurality rule. Further, the problem of ties is present too.

If we allow for *weak intersections too*, the picture doesn't change that much. For voting procedures which always result in a unique winner, there is no additional efficiency compared to the strong efficiency case. Check this for the majority rule, the runoff procedure and strict rules from Hare and Coombs in table A.2. There is only a weak intersection if a voting procedure gives tied winners. For the ties versions of Hare and Coombs there is only a marginal increase in efficiency. If there exists a Condorcet winner and the rule with ties of Hare/Coombs select that one, then it is almost always the only winner they declare. In that case, the chance for another tied winner is almost nihil. The following reasoning can be followed: we see in most of our simulations that the ties versions of Hare and Coombs give 2 alternatives in the second-last stage, then with 25 voters only one of the 2 alternatives can win. Further remark the increase in the efficiencies of the point systems, especially for the anti-plurality rule. This is all due to the large probability of tied winners these procedures have.

Analysing the *Borda efficiencies* gives another point of view. Consider tables A.3 and A.4 with strong and strong + weak efficiencies respectively. The tables give the same trivial 100 % efficiency for the case of 2 alternatives and naturally, the Borda count is 100 % Borda efficient. In all the other cases (even with weak efficiencies), 100 % efficiency is not found. This should be read as "the other voting procedures not always select the Borda winner". There are even cases where the Borda winner, the Nanson winner and the Baldwin winner are all different, though the latter two procedures are very much "Borda-based". Consider e.g. the following profile with 6 alternatives and 6 voters:

[16] This is only true for profiles without majority winner. We stated above that the majority winner (which is the Condorcet winner) is always selected by the Hare rule, but not always by the Coombs rule. However, the number of majority winners is very limited in our simulations (due to e.g. 25 or more voters), hence this effect does not play an important role.

a	b	d	e	e	f
c	c	b	a	d	c
b	e	a	b	a	d
f	f	f	c	b	a
d	d	e	f	c	b
e	a	c	d	f	e
1	1	1	1	1	1

The Borda winner is b, the Nanson winner is a and the Baldwin winner is d. The Hare winner (with ties) is even e. In general one can state that in the absence of a Condorcet winner, all voting procedures can give another winner if we let the number of alternatives increase and the probability for this increases with the number of alternatives.

The main result from the Borda efficiency tables is that the Black rule is superior. Logically, as this rule selects the Condorcet winner if she exists and in about 83 % of these cases this is the Borda winner too, else the Borda winner is directly chosen. With a larger number of alternatives, the efficiency increases as less Condorcet winners (which can be different from the Borda winner) exist.

Second, the rules from Copeland and Simpson as well as the Nanson and Baldwin procedure perform well, especially with a smaller number of alternatives. It is remarkable that the gain by taking weak intersections into account is much larger for the Simpson and Copeland procedure than for the Nanson and Baldwin procedure. Apparently, these rules give more tied winners than the Nanson and Baldwin procedure. The not-strict versions of Hare and Coombs perform rather well too. Approval voting has weak results for strong Borda efficiency for a small number of alternatives and is in between Hare and Coombs (ties versions) for a larger number of alternatives. However it gains a lot when weak efficiencies are taken into account. This is again due to a general finding: point systems are more engaged in tied winners than elimination procedures. Point systems calculate the scores once, using their points vector, and add up. This gives more equal results than elimination procedures where alternatives are eliminated in successive stages and where only in the final stage a similar system as with the points systems is used.

With an *even number of voters* some results change. Consider table A.5 with *strong Condorcet efficiencies* for 26 voters. Comparison with the 25 voters' case there are less Condorcet winners. Even with 2 alternatives, there is not always a Condorcet winner. This fact is due to ties. If in pairwise comparison alternative a beats alternative b with 13 votes and alternative b beats alternative a with 13 votes, then neither a nor b can be a Condorcet winner. In general, with an odd number of voters it is always true that the elements from the Dodgson matrix $x_{i,j} \neq x_{j,i}$ for $i \neq j$. With an even number of voters, it is possible that $x_{i,j} = x_{j,i} = n/2$ ($i \neq j$) and in that case neither alternative i nor j can be a Condorcet winner. As a result in most cases Condorcet efficiencies increase. With a lower number of Condorcet winners, the probability to select this winner if one exists increases relatively speaking. Therefore most conclusions drawn above hold for this table too, though the efficiencies are higher in general. Only exception is the majority rule performing worse with 26 voters compared to 25 voters. Two facts can explain this: first there are more voters (albeit one) and then it is more difficult to find a majority winner. Second, a 13-13 case in first votes is also possible.

Do we add weak Condorcet efficiencies, then a similar overall increase in efficiencies can be seen. The same reasoning as above can be used, but no new elements are found. See table A.6.

Concerning *Borda efficiencies*, another picture emerges. Consider table A.7 for *strong Borda efficiencies* with 26 voters. Let us make a comparison with the 25 voter's case. The Condorcet rule performs worse: there are more cases with no Condorcet winner. Plurality, anti-plurality and approval voting remain more or less the same. The majority rule performs worse because there are less majority winners as explained above. The Black rule performs extremely well: as there are less Condorcet winners it selects by definition more the Borda winner for sure. The rule of Simpson remains equal, the rule of Copeland performs better, especially with an increasing number of alternatives. The following reasoning can be followed. The Borda score from alternative i can be calculated as the sum from the elements on the i-th row of the Dodgson matrix. The Copeland rule also uses, in contrast with the Simpson rule, this kind of adding up elements of a new version of the Dodgson matrix. The Simpson rule merely takes the maximin value of the Dodgson matrix. This difference in procedure could be an explanation for the better performance of the Copeland rule here, because in absence of a Condorcet winner (which is more often the case with an even number of voters) it is more likely to select the Borda winner.

The strict rules from Hare and Coombs as well as runoff perform worse because less winners emerge with an even number of voters (larger probability for ties) and if a winner emerges, then it is a unique winner in contrast with the Borda rule selecting one or more winners. In the latter case a strong intersection is impossible. For the ties-versions of Hare and Coombs, performing slightly worse, a similar story can be told: there is always at least one winner, but a strong intersection with the Borda winner(s) does not always happen. Also the Nanson and Baldwin procedure perform worse. They always select the Condorcet winner if there exists one. Then, for all cases in which there is exactly one Borda winner, equal to the Condorcet winner, it is also the same winner of the Nanson and Baldwin procedure. For 25 voters, this happens more than for 26 voters as there are less Condorcet winners in the latter case. Apparently, in absence of a Condorcet winner, the Nanson and Baldwin winner(s) and the Borda winner(s) are mostly not exactly the same. For e.g. 3 alternatives (table A.7), there are 6.878 Condorcet winners. There is a unique Borda winner equal to the Condorcet winner in 6.566 out of these 6.878 cases. So, the strong Borda efficiency for the Baldwin and Nanson rules is 65,66 % at least and can be (10.000 - (6.878 - 6.566)) / 100 = 96,88 % at most. Of the 3.122 remaining cases, only in less than 1.000 of them the Nanson/Baldwin winner(s) are exactly the same as the Borda winner.

If we also allow for *weak intersections* (see the table A.8) then the Borda efficiency of the rules of e.g. Copeland, Simpson, Nanson, Baldwin, Hare (with ties) and Coombs (with ties) is much better: there are a lot of weak intersections between these rules' winner(s) and the winner(s) from the Borda count. Note that this effect seems to be stronger with an even number of alternatives. The effects of less Condorcet winners and more ties (e.g. in the last stage of the Hare rule with ties) may play a role here. Remark that the runoff rule and the strict versions from Hare and Coombs do not perform much better when allowing for weak intersections compared to strong intersections only. This may be due to the larger probability of ties with an even number of voters, resulting in a "no winner" case, and taking weak intersections into account does by definition not change the efficiency for those cases.

In general one can state that it is not always straightforward to come up with reasons for differences between efficiencies. The number of strong and weak intersections is for sure a function of the number of voters: an even or odd number (resulting in respectively more or less ties, less and more Condorcet winners). The number of alternatives and the ratio number of alternatives / number of voters could also be a direct parameter. However, the research for a general function defining the relationship between efficiencies and influencing parameters falls beyond the scope of this paper. As an indication how future research can be done, we calculated once the different strong Condorcet efficiencies for 5 alternatives and several numbers of voters. This is given in table A.9. One important fact is that an odd number of voters gives about the same percentage of Condorcet winners (roughly 75 %). This is not so for an even number of voters, where the percentage ranges between 50 % and 65 %. The following reasoning can be followed: with e.g. 26 voters there are less Condorcet winners than with 25 voters as explained above. For e.g. 226 voters compared to 225 voters, this holds, but to a lesser extent. This is due to the fact that e.g. a 13-13 tie is much more probable than a 113-113 tie. The disadvantage of more ties in the case of an even number of voters (with no Condorcet winner as result) declines as the number of voters increases.

Without going into details concerning these strong efficiencies, however one fact prevails for most voting procedures: oddness or evenness of the number of voters does matter more than the number itself. Exceptions are runoff and the strict rules from Hare and Coombs. Apparently, here the number itself of voters does matter most. In order to get an overall picture we should analyse tables like this for all number of alternatives. This is a topic for future research.

We can conclude that the Nanson and Baldwin rule are 100% Condorcet efficient, as shown above. The Borda rule is the best one in Condorcet efficiency of all the non-Condorcet consistent rules. The ties-versions of Hare and Coombs also perform well. In Condorcet efficiency, they perform best (rules that always select the Condorcet winner left aside) for a small number of alternatives, for a larger number of voters only the Borda count performs better. Remark that the Coombs' rule always performs better than Hare's rule.

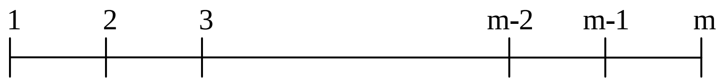
The one and only superior rule in Borda efficiency is the Black rule, which is logical by definition. Then follow the rules from Simpson and especially Copeland. The Nanson and the Baldwin rule follow then with Nanson performing slightly better. The Coombs rule (with ties) comes next, then the Hare rule (if looking at a small number of alternatives). Apparently, rules selecting the Condorcet winner if one exists are best at selecting the Borda winner.

3.2 Program for single peaked preferences

3.2.1 General analysis of single peaked preferences

The concept of single peaked preferences became very important with Black's Median Voter Theorem (1958). It gives a more realistic kind of preference orderings and has some other and more nice results even when used in randomly generated elections.

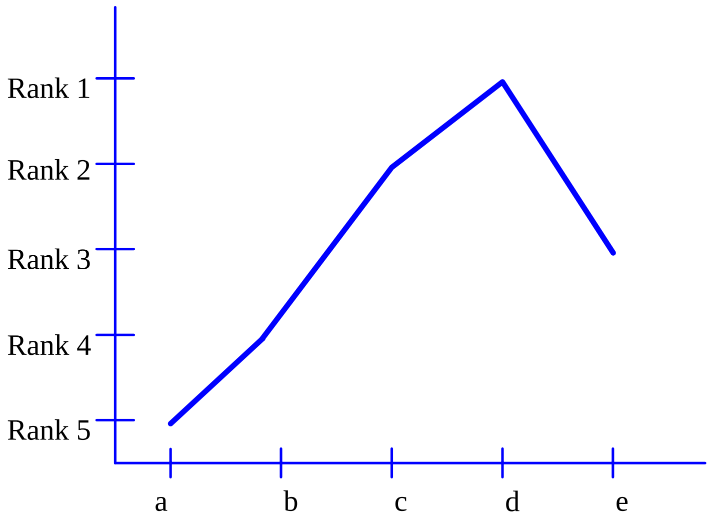
In order to define a single peaked preference we first need to rank the m alternatives on a 1-dimensional scale, e.g. according to political faith. We use the following scale:



Thereby we consider position 1 as the "extremely left" position and the position m as the "extremely right" position. Furthermore, position i, for i = 2, 3, ..., m-1, is more "left" than position i+1 and more "right" than position i-1. The complete reading of this scale of "political faith" is then straightforward. By convention we put alternative a on position 1, alternative b on position 2 and so on.

A single peaked preference ordering is one where the ranks are filled in the following "logical" way. The voter is free to put any alternative on rank 1. Then in order to fill ranks 2 to m-1 the following algorithm is followed: if rank j has to be filled in, it should be one of the two alternatives which are 1) not yet posted on a rank smaller than j and 2) located next (already posted alternatives put aside) to position i if rank j-1 was filled in by an alternative on position i. The last rank is then given to the not yet posted alternative. Note that this must be either the alternative on the extremely left position or on the extremely right position. In general we can state that in the case of m alternatives, out of the m! possible preference orderings 2^{m-1} will be single peaked. This is a general result we mention without proof. Then, with an increasing number of alternatives the ratio 2^{m-1} / m! becomes smaller and smaller. In the limit this ratio equals zero. This is important insofar as it tells us that the likelihood of single peaked preferences becomes smaller as the number of alternatives increases. Otherwise stated, if we limit the domain to single peaked preferences, then we are limiting the usable profiles in a drastic way as the number of alternatives increases.

The term single peaked preference comes from the geometrical representation of this kind of preferences. All single peaked preferences have exactly one peak as illustrated by the following graph where the single peaked preference ordering $d \succ c \succ e \succ b \succ a$ is depicted:



3.2.2 Set-up of the program for single peaked preferences

We changed the basic program slightly in order to simulate elections with "logical" voters, all revealing a single peaked preference ordering. Again the simulation starts from randomly chosen single peaked preferences.

All voting procedures, except approval voting, can work given this profile. For approval voting, recall that we used a special set-up. Here a different[17] kind of set-up was needed, with the disadvantage that the procedure is somewhat different from the previous one.

The remaining steps from the basic program are exactly the same: calculate the winner(s) for each voting procedure, do that 10.000 times and calculate the Condorcet and Borda efficiencies.

3.2.3 Results from the simulations

In the current set-up, we consider 4 cases: with 5 and 6 alternatives combined with 25 and 26 voters. Note that the difference between an even and odd number of voters again does matter. This time however, some stronger results will emerge. In the light of an analysis of these computer simulations it is important to formulate Black's Theorem here. In our terminology

[17] Before we calculated the average and used that as a cut-off point for alternatives getting a vote or not. Because of a different set-up we don't have randomly chosen real numbers corresponding to the single peaked preference orderings here, so we have to allocate numbers to the alternatives, according to their rank. Due to limitations in Pascal, it was better to use an easy solution here, such as giving m points to the alternative on rank 1, m-1 points to the one on rank 2 and so on, and then use the average (that is m*(m+1)/2) as cut-off point. With e.g. 5 alternatives, the cut-off point is (5+4+3+2+1)/5 = 3. Hence only the 2 alternatives receiving 5 and 4 points on respectively rank 1 and 2 get a vote.

and taking the conventions and other theorems from Black into account it is formulated as: “If there are n voters, all of whose preference orderings are single peaked and n is odd, then the alternative put on rank 1 (the peak) by the median voter is the Condorcet winner. If n is even, then if one and the same alternative is put on rank 1 by the 2 “median” voters, it is the Condorcet winner, if two different alternatives are put on rank 1 by the 2 “median” voters, there is no Condorcet winner.”

In order to define the *median voter(s)* we need to rank the voters on a scale according to their political faith going from an extremely left voter to an extremely right voter. Single peaked preference ordering b ≻ a ≻ c ≻ d is more “left” than e.g. single peaked preference ordering b ≻ c ≻ a ≻ d. Now write the profile as follows: put the preference ordering from a “more to the left voter” to the left of the preference ordering from a “more to the right voter”, taking equal preference orderings together. If n is odd, the median voter is the one with her preference ordering in middle of the profile. If n is even, the 2 median voters are the ones with their preference orderings in the middle of the profile.

Note that the theorem only tells us something about the possibility for a Condorcet winner, there is no relationship whatsoever with e.g. the plurality rule. The somewhat confusing “majority” concept from Black is in our terminology the Condorcet criterion. With single peaked preferences there need not to be a majority winner (in our terminology) in order to have a Condorcet winner as is shown by the following profile.

b	c	c	d
c	d	d	e
d	b	b	c
e	a	e	b
a	e	a	a
8	1	2	7

There is no majority winner, the plurality winner is b, the Condorcet winner is c and the Borda winner is d. In this example the 2 median voters have the preference orderings c ≻ d ≻ b ≻ a ≻ e and c ≻ d ≻ b ≻ e ≻ a with twice alternative c on rank 1, hence c is the Condorcet winner. In general one can state that with single peaked preferences and an odd number of voters, there is always a Condorcet winner and with an even number of voters there can be a Condorcet winner. In all cases the Borda count need not to select the Condorcet winner.

Let us first concentrate on the 2 first cases with an *odd number of voters*, i.e. 25 voters given in the 2 tables A.10 and A.11. In the tables, the efficiencies from the previous section (totally random profiles, not necessary single peaked profiles) are also given by way of comparison. Important differences between the “random” case and the “single peak” case are: in the latter case there is always a Condorcet winner, the Coombs rule (even the strict version) is 100 % Condorcet efficient, the rules from Condorcet, Black, Copeland, Simpson, Coombs, Baldwin and Nanson have exactly the same Borda efficiency and this is equal to the Condorcet efficiency of the Borda count. The Borda count is almost 100 % Condorcet efficient and the anti-plurality rule is 0 % strong Condorcet and Borda efficient, but 100 % weak Condorcet and Borda efficient. At last the runoff procedure always performs better than in the random case, while the Hare procedure does not always perform better. These results will be analysed below. Note that we cannot compare efficiencies from approval voting as a different version was programmed in the "random" and "single peak" cases.

There is always a Condorcet winner with single peaked profiles and an odd number of voters. This is directly related to Black's Theorem. This holds for our case with 25 voters.

The 100 % Condorcet efficiency for the ties version of the Coombs rule is a general result which can be proven theoretically for all number of voters, the 100 % Condorcet efficiency for the strict Coombs rule is only true for an odd number of voters. We give our proof here: it starts from the Condorcet winner and gives a reason why this alternative cannot be eliminated by the Coombs procedure.

Consider m alternatives and alternative i as Condorcet winner. This means that 1) alternative i is put on rank 1 by the median voter with n odd and by the 2 median voters with n even (Black) and 2) alternative i beats all other alternatives in pairwise comparison. We need to prove that alternative i always appears less on the last rank than any other alternative, even in a newly written profile omitting already eliminated alternatives. Using the geometrical representation of single peaked preferences, one can draw all single peaked preferences in the way described above. As the Coombs procedure eliminates the alternative(s) with the most last ranks and only the alternatives on the most left and most right position can appear on the last rank (by definition of single peaked preferences), it must be that only one or both of these 2 alternatives will be eliminated. No other alternatives can be eliminated as they appear on the second last rank or higher.

Now two cases appear at any stage where alternative i is not yet eliminated: alternative i can either be on the most left or most right position or it is on a position in between of all remaining alternatives.

In the first case, the single peaked preference ordering of the median voter(s) is a straight line. As alternative i always remains the Condorcet winner, even if other alternatives are eliminated (alternative i beats all other alternatives in pairwise comparison) we know, using Black's theorem, that alternative i appears on rank 1 of the median voter(s) of the original profile. This means that at least all voters on one "side" of the median voter(s) have alternative i as peak and the same straight line as single peaked preference ordering. Thus more than half of the voters has alternative i on rank 1, and the alternative on the other extreme position has more than half of the last rank votes and is eliminated. At any following stage, alternative i remains on the most left or most right position and the process of eliminating the alternative on the other extreme position continues. Alternative i will never be eliminated.

In the second case, the single peaked preference line is not a straight line. Other alternatives are then on the most left and most right position. Only one or both of these 2 alternatives will be eliminated by the Coombs procedure as explained above. Thus alternative i is not eliminated and in the next stage, after omitting the eliminated alternative(s), alternative i can either be on the first or last position and then we are in the first case, or not and then we are in this case again. Hence the Condorcet winner is never eliminated by the Coombs rule. End of proof.

By using the strict version of the Coombs rule and in the case of an even number of voters, it can happen that both alternatives on the first and last position have n/2 last ranks and the procedure can not declare a winner. The Condorcet winner is not eliminated (which is a general result as proven above), but there is no Coombs winner which results in a Condorcet efficiency of less than 100% in that case.

We already know that the Nanson and Baldwin rules always select the Condorcet winner (on all kinds of profiles) and also that the rules from Black, Copeland and Simpson select the Condorcet winner by definition. Hence, all these rules select the Condorcet winner as their unique winner in this case too. This is the explanation for the equal Borda efficiencies of the Condorcet, Black, Copeland, Simpson, Coombs, Baldwin and Nanson procedures. There are here 9642 cases where the Borda winner and the Condorcet winner are equal, in 163 cases the Condorcet winner is among the tied Borda winners and in the remaining 195 cases the Condorcet winner is not chosen by the Borda count.

Now, we have to explain the strange 0%-100% Condorcet/Borda efficiency from the anti-plurality rule. Recall its specific points vector (1,1,....,1,0). We know that only the 2 alternatives on the extremely left and extremely right position get last rank votes. With anti-plurality, all alternatives in between get each time 1 point. Hence, these alternatives get an equal total score and are tied winners. This explains the 0 % strong Condorcet efficiency which is generally true for $m > 2$. Take care for the other 3 efficiency results: they are due to specific factors, but do not hold in general. In our simulation with randomly composed single peaked profiles with 25 voters and 5/6 alternatives, the chance to find a particular profile where the Condorcet winner is different from all tied anti-plurality winners is nihil in practice[18], which explains the 100 % weak Condorcet efficiency.

The 0%-100% Borda efficiency of the anti-plurality rule from our simulation could also be explained by similar arguments[19].

At last, we see that runoff always performs better in terms of Condorcet and Borda efficiency with single peaked preferences. One explanation could be that there are fewer cases with ties

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[18] In theory there are some profiles with a Condorcet winner different from all tied anti-plurality winners. However, note that the probability for these case is very small.

[19] Again, a similar remark as in the previous footnote can be made.

here, as can be seen on our simulations. This may be a topic for future research. Concerning Hare, the picture is not clear at all, again to be analysed in future work.

With an *even number of voters* (26 voters) some interesting results emerge by looking at Borda efficiencies. Considering Condorcet efficiencies, no new elements are found as compared with the case of an odd number of voters (25 voters). Take a look at the next 2 tables A.12 and A.13. Remember that there is not always a Condorcet winner in this case.

By looking at the Borda efficiencies in the 2 tables A.12 and A.13, we find that on one side strong Borda efficiencies of the Simpson, Baldwin and Nanson rules and on the other side the strong + weak efficiencies of the Black, Simpson, Baldwin and Nanson rules are in both cases equal. In the case with 5 alternatives, the rules from Copeland and Coombs (with ties) give the same results as Simpson, Baldwin and Nanson. In the case with 6 alternatives, the Copeland rule has the same strong Borda efficiency as the rule of Coombs on one side and the same strong + weak efficiencies on the other side, but slightly different from the efficiencies from Simpson, Baldwin and Nanson.

These are striking results, asking for further research. The results may give an indication that with single peaked preferences the procedures from Simpson, Baldwin and Nanson always select the same winner(s) and that the procedures from Copeland and Coombs (with ties) always select the same winner(s). However, we should be careful, the results may be due to specific factors. Therefore we tested on new simulations the Condorcet, Borda, Nanson, Coombs, Copeland and Simpson efficiencies. A lot of work is still to be done in this area, but the first results in table A.14 may give an indication. Note that this table gives the results from a new simulation with 26 voters and 6 alternatives, hence the slightly different Condorcet and Borda efficiency results compared with the previous table. We also calculated tables like this one for e.g. 6 voters, 16 voters, 36 voters, 126 voters, ... but several findings seem to be robust. In all our cases, the rules from Simpson, Nanson and Baldwin give exactly the same winner(s). To a very large extent, but not always, the rules from Coombs and Copeland also select the Nanson winner(s) and almost every time they select the same winner(s). Considering strong + weak intersections, the 5 above-mentioned rules seem to have at least one winner in common.

Insofar as we can see, only the Simpson, Baldwin and Nanson rule seem to select exactly the same winners. But the Nanson winner, the Coombs winner and/or the Copeland winner need not always to be selected by the Borda count (look at the Nanson, Coombs and Copeland efficiencies of the Borda count), the Coombs rule need not always to select the Copeland winner (look at the Copeland efficiency of the Coombs rule) and so on. A lot may depend on the number of voters, the number of alternatives,

With our current set-up of the computer program we didn’t find any profile where the rules from Simpson, Baldwin and /or Nanson give another winner. The following profile with 6 voters and 6 alternatives gives another Coombs winner than the Copeland winner:

b	b	d	d	e
c	c	e	e	d
a	d	c	c	c
d	a	b	f	b
e	e	a	b	a
f	f	f	a	f
1	2	1	1	1

The Nanson, Baldwin and Simpson winners equal the set {b, c, d}, the Borda winner is d, there is no Condorcet winner, the Copeland winner is d and the Coombs (with ties) winner is c.

To conclude, we found that the ties version of the Coombs rule is 100 % Condorcet efficient with single peaked profiles, which can be proven theoretically. The strict version of Coombs has the same characteristic with an odd number of voters. Concerning Borda efficiency, the Coombs, Nanson and Baldwin rules perform much better than in the random case. Apart from the Black rule, these 3 rules are very well performing in the single peaked profile case. A finding which may be interesting for future research is that the Nanson, Baldwin and Simpson procedures seem always to select the same winners in the case of single peaked preferences, even in absence of a Condorcet winner.

4. Conclusion

In this paper 16 voting procedures for one-seat elections were analysed. In the second section the procedures were explained and the most common features were given. Throughout the paper, the difference between a voting procedure allowing for ties or not was quite crucial. The former kind of procedures is much better than the latter kind.

In the third section, we ran computer simulations in order to calculate Condorcet and Borda efficiencies. We found that the Borda rule is about 85% Condorcet efficient while some voting procedures are always 100 % Condorcet efficient (Black, Copeland, Simpson, Baldwin and Nanson). A special result in the single peaked profiles case is that the ties version of the Coombs rule is 100 % Condorcet efficient, which was also proven theoretically. The strict version of Coombs is only 100 % Condorcet efficient with an odd number of voters. Another feature from single peaked profiles seems to be that some voting procedures select the same winner (Simpson, Baldwin and Nanson). This result may be interesting for future research. Considering Borda efficiencies we see that the Black rule scores well, followed by the rules of Copeland, Nanson and Baldwin. Taking both Condorcet and Borda efficiencies into account, we can state that the Black rule is superior. Then come the rules of Borda, Copeland, Simpson, Nanson and Baldwin. All the other rules are inferior in Condorcet and Borda efficiency to the latter ones.

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Appendix

Proof of the 100 % Condorcet efficiency of the Nanson and Baldwin rules

Remember that the sum of the elements on the i-th row of the Dodgson matrix gives the Borda score for alternative i. Further we saw above that the total number of Borda points given to the m alternatives by the n voters equals $n * (m-1) * m / 2$. The average number of Borda points per alternative equals then $n * (m-1) / 2$. Now consider two cases: n is even and n is odd. If n is even and alternative j is the Condorcet winner, then each element (except the diagonal element that is equal to 0) of the j-th row of the Dodgson matrix equals $n/2 + 1$ at least as j wins all pairwise comparisons by 1 vote at least. The sum of these (m-1) elements equals minimally $(m - 1) * (n / 2 + 1) = ((m-1) * n / 2) + m - 1$. This is the minimal Borda score for alternative j, and is higher than the average Borda score per alternative. Hence, there is at least one alternative with a lower than average Borda score than alternative j. So alternative j cannot be eliminated by the Nanson/Baldwin rule. This result is always true: if k alternatives are already eliminated, then the same minimal Borda score for Condorcet winner j and the same average Borda score per voter emerge if we just replace m by (m - k). If n is odd, not so much changes: each element (except the diagonal element) of the j-th row of the Dodgson matrix equals $n/2 + 1/2$ at least and the minimal Borda score for the Condorcet winner j equals $((m-1) * n / 2) + ½ (m - 1)$. The same conclusion can be drawn. The Condorcet winner can never be eliminated by the Nanson and Baldwin rules and is declared as the winner in the final stage. The Nanson and Baldwin procedures are 100 % Condorcet efficient.

TABLE A.1 : RANDOM PROFILE, 25 VOTERS, STRONG CONDORCET EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	87.85	84.47	83.04	83.19	82.23	80.81	82.41	82.41	82.35	81.86	82.51	83.44	83.78	83.77	82.50	84.35	83.81	83.57
CONDORCET	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
PLURALITY	100.00	73.63	62.25	54.24	48.36	42.75	39.14	36.49	34.36	31.19	29.19	26.56	25.44	24.23	24.31	22.56	20.20	19.89	19.33
MAJORITY	100.00	13.73	1.62	0.24	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	66.78	51.21	41.10	32.97	26.66	21.53	17.56	15.19	12.69	8.94	7.08	4.15	2.32	0.89	0.32	0.11	0.09	0.00
APPROVAL	100.00	70.38	62.75	59.67	57.57	55.32	53.40	54.17	53.48	52.32	52.58	50.99	51.14	50.89	50.51	49.39	49.76	50.03	49.52
BLACK	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
COPELAND	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
SIMPSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
HARE	100.00	82.29	60.16	37.75	19.92	7.17	2.06	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	85.63	68.35	49.83	34.46	22.92	14.44	8.55	4.59	2.33	1.23	0.74	0.26	0.10	0.10	0.03	0.00	0.00	0.00
RUNOFF	100.00	84.63	71.99	62.42	52.75	47.79	42.98	38.71	35.60	33.88	29.87	26.86	24.45	23.16	20.78	21.43	19.55	20.56	20.14
HARE (TIES)	100.00	93.89	89.48	85.12	81.66	78.46	77.21	73.89	72.86	70.40	67.54	65.62	66.03	64.05	63.14	62.69	59.82	59.58	58.65
COOMBS (TIES)	100.00	94.82	91.44	87.08	84.94	83.00	80.18	79.66	79.05	77.06	73.47	74.03	74.79	71.69	70.94	70.01	69.14	68.08	67.09
BALDWIN	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
NANSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
% Condorcet winners	100.00	91.82	84.07	75.52	70.43	63.88	60.19	55.31	52.47	50.21	47.07	45.75	42.45	41.36	39.12	37.15	35.39	34.34	33.42

TABLE A.2 : RANDOM PROFILE, 25 VOTERS, STRONG + WEAK CONDORCET EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	94.34	90.31	88.08	87.28	86.51	84.43	85.48	85.48	85.22	84.24	85.03	85.77	85.78	84.89	84.28	85.90	85.32	84.95
CONDORCET	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
PLURALITY	100.00	84.50	76.76	69.92	64.79	60.10	58.02	54.64	53.02	49.91	47.44	46.12	45.14	43.67	43.05	42.05	39.67	40.24	39.02
MAJORITY	100.00	13.73	1.62	0.24	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	80.07	67.32	60.59	54.66	49.70	47.33	45.69	44.69	43.14	41.62	41.07	41.15	42.21	45.27	48.18	50.44	54.25	54.88
APPROVAL	100.00	82.86	77.54	76.06	74.03	72.35	71.96	71.49	71.36	70.21	69.79	70.08	69.92	69.61	69.79	68.75	70.08	70.06	68.73
BLACK	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
COPELAND	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
SIMPSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
HARE	100.00	82.29	60.16	37.75	19.92	7.17	2.06	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	85.63	68.35	49.83	34.46	22.92	14.44	8.55	4.59	2.33	1.23	0.74	0.26	0.10	0.10	0.03	0.00	0.00	0.00
RUNOFF	100.00	84.63	71.99	62.42	52.75	47.79	42.98	38.71	35.60	33.88	29.87	26.86	24.45	23.16	20.78	21.43	19.55	20.56	20.14
HARE (TIES)	100.00	93.89	89.48	85.29	81.84	78.85	77.54	74.31	73.17	70.74	67.77	66.19	66.34	64.48	63.62	63.10	60.02	60.10	58.95
COOMBS (TIES)	100.00	94.82	91.44	87.24	85.15	83.25	80.41	79.90	79.42	77.34	73.59	74.30	74.86	71.91	71.14	70.12	69.40	68.32	67.27
BALDWIN	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
NANSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
% Condorcet winners	100.00	91.82	84.07	75.52	70.43	63.88	60.19	55.31	52.47	50.21	47.07	45.75	42.45	41.36	39.12	37.15	35.39	34.34	33.42

TABLE A.3 : RANDOM PROFILE, 25 VOTERS, STRONG BORDA EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
CONDORCET	100.00	80.66	71.01	62.71	58.59	52.53	48.64	45.58	43.24	41.35	38.53	37.75	35.42	34.65	32.77	30.65	29.85	28.78	27.93
PLURALITY	100.00	69.54	55.75	46.87	40.74	35.05	31.32	27.88	25.91	22.84	21.55	19.49	18.31	17.66	16.52	15.35	14.29	13.35	12.32
MAJORITY	100.00	12.54	1.36	0.18	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	66.07	50.59	40.27	31.97	25.97	21.02	17.23	14.45	12.20	8.98	6.74	4.08	2.31	1.21	0.40	0.15	0.03	0.02
APPROVAL	100.00	68.34	60.55	56.59	53.84	50.86	49.38	49.05	47.53	46.37	46.06	44.27	43.76	42.95	42.96	41.48	40.91	41.27	40.58
BLACK	100.00	88.84	86.94	87.19	88.16	88.65	88.45	90.27	90.77	91.14	91.46	92.00	92.97	93.29	93.65	93.50	94.46	94.44	94.51
COPELAND	100.00	81.68	71.90	66.06	64.38	61.51	56.89	59.20	59.13	58.42	58.08	57.94	57.90	57.81	57.66	57.35	57.89	57.08	57.27
SIMPSON	100.00	83.83	75.78	69.83	67.03	62.41	59.88	58.23	55.90	55.03	53.36	52.68	51.42	50.86	49.48	48.29	47.94	47.31	46.87
HARE	100.00	68.11	45.72	26.83	13.55	4.86	1.29	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	71.48	53.27	36.41	25.03	15.40	9.67	5.44	2.79	1.50	0.83	0.45	0.15	0.04	0.04	0.02	0.00	0.00	0.00
RUNOFF	100.00	70.25	55.83	46.26	38.77	33.17	29.42	26.64	24.73	22.28	19.74	18.08	15.52	14.94	13.29	13.47	12.80	12.68	12.45
HARE (TIES)	100.00	79.12	70.49	63.50	59.62	55.38	52.24	49.88	48.46	46.21	43.81	42.19	40.75	40.14	38.09	37.26	36.18	36.13	34.96
COOMBS (TIES)	100.00	80.68	73.70	67.19	65.66	60.90	59.57	57.52	55.82	54.50	52.12	52.26	51.70	50.03	49.56	48.23	47.36	46.30	45.66
BALDWIN	100.00	83.60	75.59	70.76	68.38	64.47	62.76	61.63	59.55	58.80	57.21	56.74	55.08	55.68	54.21	53.34	52.62	52.21	51.26
NANSON	100.00	83.83	76.66	71.39	69.54	66.20	64.41	63.12	61.01	60.54	59.07	58.27	57.22	57.06	56.16	55.12	55.19	54.16	53.62
% Condorcet winners	100.00	91.82	84.07	75.52	70.43	63.88	60.19	55.31	52.47	50.21	47.07	45.75	42.45	41.36	39.12	37.15	35.39	34.34	33.42

TABLE A.4 : RANDOM PROFILE, 25 VOTERS, STRONG + WEAK BORDA EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
CONDORCET	100.00	86.62	75.92	66.52	61.47	55.26	50.82	47.28	44.85	42.79	39.65	38.90	36.41	35.48	33.21	31.31	30.40	29.30	28.39
PLURALITY	100.00	85.73	75.70	67.38	60.61	55.57	52.27	47.91	45.84	42.71	40.63	38.78	37.06	36.61	34.03	33.53	31.94	31.67	29.64
MAJORITY	100.00	12.56	1.36	0.18	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	84.93	71.91	64.26	57.11	51.84	49.46	46.53	45.17	44.11	41.91	41.75	41.40	43.19	44.51	47.98	50.12	52.90	54.78
APPROVAL	100.00	86.20	80.67	78.27	75.29	73.34	72.35	71.24	70.37	68.60	68.55	67.27	66.82	66.27	65.69	65.23	65.02	65.35	65.44
BLACK	100.00	94.80	91.85	91.00	91.04	91.38	90.63	91.97	92.38	92.58	92.58	93.15	93.96	94.12	94.09	94.16	95.01	94.96	94.97
COPELAND	100.00	94.80	91.47	89.22	87.67	86.59	84.21	84.00	83.25	82.62	82.23	81.49	81.67	80.92	79.80	79.36	79.71	78.74	79.15
SIMPSON	100.00	94.52	90.30	87.31	85.54	84.39	82.49	82.55	81.57	80.19	79.59	79.35	78.77	78.69	77.00	76.91	77.05	76.75	76.56
HARE	100.00	74.19	49.86	29.25	14.61	5.27	1.42	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	77.97	57.85	39.69	26.77	16.53	10.27	5.81	3.02	1.54	0.84	0.45	0.18	0.05	0.04	0.02	0.00	0.00	0.00
RUNOFF	100.00	76.34	60.93	50.35	41.63	36.00	31.71	28.49	26.58	23.70	20.85	19.20	16.51	15.89	13.82	14.12	13.29	13.39	13.02
HARE (TIES)	100.00	86.55	77.05	69.51	64.51	60.72	56.68	54.05	52.38	49.58	46.73	45.10	43.57	42.67	40.00	39.53	38.09	38.12	36.69
COOMBS (TIES)	100.00	88.25	80.43	73.53	70.83	66.35	64.00	61.81	60.07	58.00	55.22	55.42	54.51	52.60	51.90	50.70	49.56	48.31	47.88
BALDWIN	100.00	90.23	83.97	79.20	75.38	72.50	69.98	68.48	66.47	65.48	63.86	62.93	61.19	61.04	59.59	58.93	58.25	57.99	56.52
NANSON	100.00	90.46	84.51	79.56	76.05	73.43	70.73	69.41	67.18	66.30	64.65	63.91	62.24	61.77	60.59	59.61	59.38	58.66	57.80
% Condorcet winners	100.00	91.82	84.07	75.52	70.43	63.88	60.19	55.31	52.47	50.21	47.07	45.75	42.45	41.36	39.12	37.15	35.39	34.34	33.42

TABLE A.5 : RANDOM PROFILE, 26 VOTERS, STRONG CONDORCET EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	95.46	93.77	92.69	92.11	91.90	91.64	92.90	91.87	92.05	91.92	92.16	91.77	92.78	92.94	93.11	92.37	92.22	92.54
CONDORCET	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
PLURALITY	100.00	81.14	69.90	60.94	55.13	50.11	47.20	41.37	39.66	37.03	34.70	33.31	29.87	29.12	27.43	26.04	25.09	23.74	21.58
MAJORITY	100.00	10.42	0.97	0.28	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	73.04	59.21	46.57	36.84	29.44	25.14	21.70	16.47	13.81	10.92	7.75	5.38	2.76	1.06	0.56	0.40	0.06	0.19
APPROVAL	100.00	77.30	72.18	68.37	66.31	66.36	64.77	63.73	62.05	62.67	61.08	61.10	60.57	62.11	62.93	60.97	58.44	60.35	61.38
BLACK	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
COPELAND	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
SIMPSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
HARE	100.00	85.55	63.80	41.01	21.96	9.73	2.57	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	86.87	69.90	52.88	36.82	25.11	14.83	9.07	4.91	2.64	2.21	0.59	0.41	0.39	0.15	0.00	0.00	0.00	0.00
RUNOFF	100.00	87.48	75.62	65.46	57.92	52.71	47.29	42.53	40.74	38.57	33.99	32.92	28.58	27.03	24.61	24.82	22.45	23.50	20.71
HARE (TIES)	100.00	97.38	94.14	90.99	87.73	86.73	84.18	81.39	81.38	79.52	76.62	76.02	72.61	71.95	71.61	71.29	68.89	65.91	65.17
COOMBS (TIES)	100.00	96.61	94.01	91.53	90.15	87.93	85.96	84.37	83.19	82.20	81.39	79.45	79.04	80.33	78.37	77.46	76.06	74.22	74.07
BALDWIN	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
NANSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
% Condorcet winners	84.64	68.78	59.06	50.08	44.08	39.27	35.40	31.98	29.75	27.30	25.36	23.60	21.76	20.64	19.83	18.01	17.42	16.72	16.08

TABLE A.6 : RANDOM PROFILE, 26 VOTERS, STRONG + WEAK CONDORCET EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	98.44	96.63	95.39	94.62	94.42	93.36	94.50	93.45	93.63	93.45	93.18	92.88	93.65	94.20	94.06	93.57	93.48	92.97
CONDORCET	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
PLURALITY	100.00	90.68	82.68	75.48	71.12	66.26	64.38	60.07	58.66	56.08	53.79	52.16	49.59	48.11	46.95	45.92	45.46	43.84	41.73
MAJORITY	100.00	10.42	0.97	0.28	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	84.08	75.13	65.73	59.53	54.11	51.21	48.97	47.16	46.08	44.01	43.81	44.67	46.12	48.11	49.97	52.30	54.25	57.59
APPROVAL	100.00	87.44	84.13	82.33	80.63	80.19	79.77	79.24	78.86	78.94	77.60	78.01	77.07	79.17	79.22	79.46	76.75	79.07	78.17
BLACK	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
COPELAND	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
SIMPSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
HARE	100.00	85.55	63.80	41.01	21.96	9.73	2.57	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	86.87	69.90	52.88	36.82	25.11	14.83	9.07	4.91	2.64	2.21	0.59	0.41	0.39	0.15	0.00	0.00	0.00	0.00
RUNOFF	100.00	87.48	75.62	65.46	57.92	52.71	47.29	42.53	40.74	38.57	33.99	32.92	28.58	27.03	24.61	24.82	22.45	23.50	20.71
HARE (TIES)	100.00	97.38	94.14	90.99	87.73	86.73	84.18	81.39	81.38	79.52	76.62	76.02	72.61	71.95	71.61	71.29	68.89	65.91	65.17
COOMBS (TIES)	100.00	96.61	94.01	91.53	90.15	87.93	85.96	84.37	83.19	82.20	81.39	79.45	79.04	80.33	78.37	77.46	76.06	74.22	74.07
BALDWIN	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
NANSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
% Condorcet winners	84.64	68.78	59.06	50.08	44.08	39.27	35.40	31.98	29.75	27.30	25.36	23.60	21.76	20.64	19.83	18.01	17.42	16.72	16.08

TABLE A.7 : RANDOM PROFILE, 26 VOTERS, STRONG BORDA EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
CONDORCET	100.00	65.66	55.38	46.42	40.60	36.09	32.44	29.71	27.33	25.13	23.31	21.75	19.97	19.15	18.43	16.77	16.09	15.42	14.88
PLURALITY	100.00	68.17	56.55	47.02	40.97	35.65	32.28	28.05	25.58	23.79	21.59	20.74	18.44	17.08	16.33	15.09	13.83	13.49	12.39
MAJORITY	100.00	7.16	0.57	0.14	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	65.59	51.01	40.22	32.08	25.89	21.46	17.84	14.38	12.20	9.90	7.14	4.86	2.78	1.42	0.63	0.31	0.08	0.04
APPROVAL	100.00	67.41	61.21	56.98	53.46	51.68	49.93	48.65	47.59	46.81	45.81	44.64	43.29	43.42	43.87	41.79	41.54	40.57	40.80
BLACK	100.00	96.88	96.32	96.34	96.52	96.82	97.04	97.73	97.58	97.83	97.95	98.15	98.21	98.51	98.60	98.76	98.67	98.70	98.80
COPELAND	100.00	82.78	76.44	73.17	71.32	69.78	68.90	68.68	68.24	67.43	67.62	68.15	68.17	66.61	68.00	66.67	66.47	66.86	67.04
SIMPSON	100.00	77.60	70.47	65.05	62.12	59.26	57.28	55.76	54.38	53.07	52.16	51.65	50.59	48.78	49.93	47.32	46.64	46.85	46.08
HARE	100.00	60.49	41.39	24.68	12.34	4.98	1.47	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	62.10	46.22	31.96	21.28	13.54	8.05	4.60	2.50	1.35	0.81	0.30	0.20	0.11	0.10	0.00	0.00	0.00	0.00
RUNOFF	100.00	61.82	50.04	41.72	34.84	30.54	26.98	24.38	22.10	20.88	18.32	17.47	15.18	13.17	12.45	12.18	10.97	11.42	10.44
HARE (TIES)	100.00	72.79	65.34	59.13	53.79	50.39	47.71	45.73	43.41	42.23	40.72	39.39	36.82	35.80	35.23	33.93	33.37	31.87	30.94
COOMBS (TIES)	100.00	74.47	67.83	61.81	59.19	55.72	52.87	51.84	49.60	47.79	47.97	46.34	45.59	44.62	43.84	43.33	41.76	41.55	41.44
BALDWIN	100.00	76.39	68.19	61.54	57.64	55.05	52.95	51.73	49.57	48.39	47.48	46.47	45.64	44.12	43.97	42.62	42.24	41.94	41.41
NANSON	100.00	77.60	68.99	62.87	59.43	56.69	54.70	52.87	51.34	49.85	49.22	48.35	47.57	45.99	45.57	44.36	44.45	43.48	43.01
% Condorcet winners	84.64	68.78	59.06	50.08	44.08	39.27	35.40	31.98	29.75	27.30	25.36	23.60	21.76	20.64	19.83	18.01	17.42	16.72	16.08

TABLE A.8 : RANDOM PROFILE, 26 VOTERS, STRONG + WEAK BORDA EFFICIENCY (IN %)

procedure \ # alternatives	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
BORDA	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
CONDORCET	100.00	67.71	57.07	47.77	41.71	37.08	33.05	30.22	27.80	25.56	23.70	21.99	20.21	19.33	18.68	16.94	16.30	15.63	14.95
PLURALITY	100.00	86.57	76.48	67.07	61.52	55.69	52.23	48.07	45.19	43.03	40.51	38.66	36.61	35.12	34.17	32.64	31.31	30.52	29.21
MAJORITY	100.00	7.17	0.57	0.14	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTI-PLURALTY	100.00	83.37	72.62	63.01	57.71	52.25	49.27	45.34	44.91	42.88	41.85	40.43	40.29	42.12	43.83	45.86	47.86	50.61	52.24
APPROVAL	100.00	84.83	80.12	76.93	74.57	72.89	71.52	70.87	69.35	69.48	68.50	67.87	66.26	66.57	66.43	65.21	64.40	64.58	63.89
BLACK	100.00	98.93	98.01	97.69	97.63	97.81	97.65	98.24	98.05	98.26	98.34	98.39	98.45	98.69	98.85	98.63	98.88	98.61	98.87
COPELAND	100.00	97.86	94.70	91.57	89.34	87.63	86.21	85.22	84.09	83.52	83.17	82.91	82.40	81.20	81.80	80.68	80.00	80.26	79.75
SIMPSON	100.00	97.80	94.45	90.91	87.75	86.34	84.89	83.48	82.71	81.17	80.77	81.21	80.07	79.20	79.10	78.00	77.36	77.62	76.43
HARE	100.00	64.59	44.39	26.41	13.28	5.27	1.52	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COOMBS	100.00	65.58	49.15	33.82	22.67	14.37	8.42	4.82	2.59	1.43	0.88	0.33	0.20	0.11	0.10	0.00	0.00	0.00	0.00
RUNOFF	100.00	65.92	53.54	44.54	37.28	32.73	28.63	25.72	23.36	21.88	19.37	18.17	16.01	13.77	13.18	12.69	11.61	11.91	10.89
HARE (TIES)	100.00	92.32	84.40	76.59	70.10	65.58	62.28	58.95	55.77	54.26	51.84	50.04	47.51	45.86	44.79	43.41	41.79	40.13	39.39
COOMBS (TIES)	100.00	91.73	84.86	78.35	74.22	70.53	66.89	65.29	62.97	60.07	60.02	58.21	56.68	55.78	54.94	54.03	51.60	51.81	50.97
BALDWIN	100.00	96.43	92.23	87.65	83.92	81.26	79.30	77.62	76.05	73.95	73.07	73.39	71.53	69.96	69.78	68.90	67.19	67.00	66.49
NANSON	100.00	96.48	92.51	87.85	84.13	81.66	79.58	77.95	76.56	74.57	73.63	73.95	71.87	70.43	70.42	69.42	67.95	67.69	67.02
% Condorcet winners	84.64	68.78	59.06	50.08	44.08	39.27	35.40	31.98	29.75	27.30	25.36	23.60	21.76	20.64	19.83	18.01	17.42	16.72	16.08

TABLE A.9: RANDOM PROFILE, 5 ALTERNATIVES, STRONG CONDORCET EFFICIENCY (IN %)

procedure \ # voters	25	75	125	225	26	76	126	226
BORDA	83.04	84.55	85.06	84.69	92.69	90.58	88.57	88.32
CONDORCET	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
PLURALITY	54.24	55.61	55.19	54.64	60.94	59.10	58.73	59.25
MAJORITY	0.24	0.00	0.00	0.00	0.28	0.00	0.00	0.00
ANTI-PLURALTY	41.10	47.84	49.71	51.31	46.57	50.37	53.03	52.41
APPROVAL	59.67	63.67	65.01	65.30	68.37	68.22	68.74	68.42
BLACK	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
COPELAND	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
SIMPSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
HARE	37.75	58.74	65.71	70.99	41.01	59.78	66.15	72.53
COOMBS	49.83	63.31	67.34	72.58	52.88	65.41	70.28	74.50
RUNOFF	62.42	69.12	71.12	73.65	65.46	71.04	72.77	75.63
HARE (TIES)	85.12	88.14	87.86	88.32	90.99	90.34	90.19	90.24
COOMBS (TIES)	87.08	87.47	88.72	88.57	91.53	91.60	90.70	90.73
BALDWIN	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
NANSON	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
% Condorcet winners	75.52	75.52	74.31	75.36	50.08	58.78	62.22	65.48

TABLE A.10: RANDOM & SINGLE PEAKED PROFILE, 25 VOTERS, 5 ALTERNATIVES, EFFICIENCIES IN %

	Condorcet efficiency				Borda efficiency			
	strong		strong + weak		strong		strong + weak	
	random	**single peak**	random	**single peak**	random	**single peak**	random	**single peak**
BORDA	83.04	**96.42**	88.08	**98.05**	100.00	**100.00**	100.00	**100.00**
CONDORCET	100.00	**100.00**	100.00	**100.00**	62.71	**96.42**	66.52	**98.05**
PLURALITY	54.24	**65.36**	69.92	**76.41**	46.87	**62.47**	67.38	**75.16**
MAJORITY	0.24	**8.81**	0.24	**8.81**	0.18	**8.62**	0.18	**8.81**
ANTI-PLURALTY	41.10	**0.00**	60.59	**100.00**	40.27	**0.00**	64.26	**100.00**
APPROVAL	59.67	**94.31**	76.06	**95.89**	56.59	**94.80**	78.27	**98.01**
BLACK	100.00	**100.00**	100.00	**100.00**	87.19	**96.42**	91.00	**98.05**
COPELAND	100.00	**100.00**	100.00	**100.00**	66.06	**96.42**	89.22	**98.05**
SIMPSON	100.00	**100.00**	100.00	**100.00**	69.83	**96.42**	87.31	**98.05**
HARE	37.75	**47.10**	37.75	**47.10**	26.83	**45.16**	29.25	**46.06**
COOMBS	49.83	**100.00**	49.83	**100.00**	36.41	**96.42**	39.69	**98.05**
RUNOFF	62.42	**79.01**	62.42	**79.01**	46.26	**75.97**	50.35	**77.42**
HARE (TIES)	85.12	**77.62**	85.29	**77.62**	63.50	**74.22**	69.51	**75.85**
COOMBS (TIES)	87.08	**100.00**	87.24	**100.00**	67.19	**96.42**	73.53	**98.05**
BALDWIN	100.00	**100.00**	100.00	**100.00**	70.76	**96.42**	79.20	**98.05**
NANSON	100.00	**100.00**	100.00	**100.00**	71.39	**96.42**	79.56	**98.05**
% Condorcet winners	75.52	**100.00**	75.52	**100.00**	75.52	**100.00**	75.52	**100.00**

TABLE A.11: RANDOM & SINGLE PEAKED PROFILE, 25 VOTERS, 6 ALTERNATIVES, EFFICIENCIES IN %

	Condorcet efficiency				Borda efficiency			
	strong		strong + weak		strong		strong + weak	
	random	**single peak**	random	**single peak**	random	**single peak**	random	**single peak**
BORDA	83.19	**86.13**	87.28	**91.32**	100.00	**100.00**	100.00	**100.00**
CONDORCET	100.00	**100.00**	100.00	**100.00**	58.59	**86.13**	61.47	**91.32**
PLURALITY	48.36	**71.25**	64.79	**80.99**	40.74	**70.73**	60.61	**84.00**
MAJORITY	0.06	**4.86**	0.06	**4.86**	0.04	**4.86**	0.04	**4.86**
ANTI-PLURALTY	32.97	**0.00**	54.66	**100.00**	31.97	**0.00**	57.11	**100.00**
APPROVAL	57.57	**60.18**	74.03	**74.81**	53.84	**59.91**	75.29	**77.87**
BLACK	100.00	**100.00**	100.00	**100.00**	88.16	**86.13**	91.04	**91.32**
COPELAND	100.00	**100.00**	100.00	**100.00**	64.38	**86.13**	87.67	**91.32**
SIMPSON	100.00	**100.00**	100.00	**100.00**	67.03	**86.13**	85.54	**91.32**
HARE	19.92	**29.53**	19.92	**29.53**	13.55	**26.81**	14.61	**28.97**
COOMBS	34.46	**100.00**	34.46	**100.00**	25.03	**86.13**	26.77	**91.32**
RUNOFF	52.75	**79.83**	52.75	**79.83**	38.77	**69.75**	41.63	**73.86**
HARE (TIES)	81.66	**83.42**	81.84	**83.42**	59.62	**74.58**	64.51	**79.77**
COOMBS (TIES)	84.94	**100.00**	85.15	**100.00**	65.66	**86.11**	70.83	**91.32**
BALDWIN	100.00	**100.00**	100.00	**100.00**	68.38	**86.13**	75.38	**91.32**
NANSON	100.00	**100.00**	100.00	**100.00**	69.54	**86.13**	76.05	**91.32**
% Condorcet winners	70.43	**100.00**	70.43	**100.00**	70.43	**100.00**	70.43	**100.00**

TABLE A.12: RANDOM & SINGLE PEAKED PROFILE, 26 VOTERS, 5 ALTERNATIVES, EFFICIENCIES IN %

	Condorcet efficiency				Borda efficiency			
	strong		strong + weak		strong		strong + weak	
	random	**single peak**	random	**single peak**	random	**single peak**	random	**single peak**
BORDA	92.69	**99.10**	95.39	**99.29**	100.00	**100.00**	100.00	**100.00**
CONDORCET	100.00	**100.00**	100.00	**100.00**	46.42	**94.06**	47.77	**94.24**
PLURALITY	60.94	**69.73**	75.48	**78.76**	47.02	**66.55**	67.07	**76.18**
MAJORITY	0.28	**6.07**	0.28	**6.07**	0.14	**5.76**	0.14	**5.76**
ANTI-PLURALTY	46.57	**0.00**	65.73	**100.00**	40.22	**0.00**	63.01	**100.00**
APPROVAL	68.37	**95.10**	82.33	**97.07**	56.98	**93.77**	76.93	**96.54**
BLACK	100.00	**100.00**	100.00	**100.00**	96.34	**99.15**	97.69	**99.33**
COPELAND	100.00	**100.00**	100.00	**100.00**	73.17	**94.60**	91.57	**99.33**
SIMPSON	100.00	**100.00**	100.00	**100.00**	65.05	**94.60**	90.91	**99.33**
HARE	41.01	**49.49**	41.01	**49.49**	24.68	**46.79**	26.41	**46.96**
COOMBS	52.88	**69.69**	52.88	**69.69**	31.96	**65.47**	33.82	**65.47**
RUNOFF	65.46	**77.73**	65.46	**77.73**	41.72	**73.24**	44.54	**73.42**
HARE (TIES)	90.99	**80.00**	90.99	**80.00**	59.13	**75.77**	76.59	**78.91**
COOMBS (TIES)	91.53	**100.00**	91.53	**100.00**	61.81	**94.60**	78.35	**99.33**
BALDWIN	100.00	**100.00**	100.00	**100.00**	61.54	**94.60**	87.65	**99.33**
NANSON	100.00	**100.00**	100.00	**100.00**	62.87	**94.60**	87.85	**99.33**
% Condorcet winners	50.08	**94.91**	50.08	**94.91**	50.08	**94.91**	50.08	**94.91**

TABLE A.13: RANDOM & SINGLE PEAKED PROFILE, 26 VOTERS, 6 ALTERNATIVES, EFFICIENCIES IN %

	Condorcet efficiency				Borda efficiency			
	strong		strong + weak		strong		strong + weak	
	random	single peak	random	single peak	random	single peak	random	single peak
BORDA	92.11	**91.98**	94.62	**95.35**	100.00	**100.00**	100.00	**100.00**
CONDORCET	100.00	**100.00**	100.00	**100.00**	40.60	**76.15**	41.71	**78.94**
PLURALITY	55.13	**78.05**	71.12	**86.13**	40.97	**70.35**	61.52	**83.57**
MAJORITY	0.02	**4.17**	0.02	**4.17**	0.01	**3.45**	0.01	**3.45**
ANTI-PLURALTY	36.84	**0.00**	59.53	**100.00**	32.08	**0.00**	57.71	**100.00**
APPROVAL	66.31	**59.95**	80.63	**76.69**	53.46	**57.63**	74.57	**78.00**
BLACK	100.00	**100.00**	100.00	**100.00**	96.52	**93.36**	97.63	**96.15**
COPELAND	100.00	**100.00**	100.00	**100.00**	71.32	**78.78**	89.34	**96.01**
SIMPSON	100.00	**100.00**	100.00	**100.00**	62.12	**78.64**	87.75	**96.15**
HARE	21.96	**31.83**	21.96	**31.83**	12.34	**25.73**	13.28	**26.91**
COOMBS	36.82	**70.75**	36.82	**70.50**	21.28	**53.62**	22.67	**55.47**
RUNOFF	57.92	**79.99**	57.92	**79.99**	34.84	**62.68**	37.28	**65.75**
HARE (TIES)	87.73	**87.60**	87.73	**87.60**	53.79	**72.95**	70.10	**87.15**
COOMBS (TIES)	90.15	**100.00**	90.15	**100.00**	59.19	**78.78**	74.22	**96.01**
BALDWIN	100.00	**100.00**	100.00	**100.00**	57.64	**78.64**	83.92	**96.15**
NANSON	100.00	**100.00**	100.00	**100.00**	59.43	**78.64**	84.13	**96.15**
% Condorcet winners	44.08	**82.79**	44.08	**82.79**	44.08	**82.79**	44.08	**82.79**

TABLE A.14: SINGLE PEAKED PROFILES, 26 VOTERS, 6 ALTERNATIVES, EFFICIENCIES IN %

	Condorcet efficiency		Borda efficiency		Nanson efficiency		Coombs efficiency		Copeland efficiency		Simpson efficiency	
	strong	strong+weak	strong	strong+weak	strong	strong+weak	strong	strong+weak	strong	strong+weak	strong	strong+weak
BORDA	90.94	94.79	100.00	100.00	78.86	95.62	78.99	95.58	79.04	95.58	78.86	95.62
CONDORCET	100.00	100.00	76.45	79.69	84.07	84.07	84.07	84.07	84.07	84.07	84.07	84.07
PLURALITY	75.82	84.99	68.98	82.95	66.46	86.99	66.46	86.86	66.46	86.82	66.46	86.99
MAJORITY	3.31	3.31	2.74	2.74	2.78	2.78	2.78	2.78	2.78	2.78	2.78	2.78
ANTI-PLURALTY	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00
APPROVAL	61.68	76.38	60.03	79.10	54.72	80.14	54.90	80.14	54.94	80.14	54.72	80.14
BLACK	100.00	100.00	92.38	95.62	86.48	100.00	86.61	99.96	86.65	99.96	86.48	100.00
COPELAND	100.00	100.00	79.04	95.58	99.78	100.00	99.95	100.00	100.00	100.00	99.78	100.00
SIMPSON	100.00	100.00	78.86	95.62	100.00	100.00	99.83	100.00	99.78	100.00	100.00	100.00
HARE	30.58	30.58	25.10	26.28	25.71	27.26	25.71	27.26	25.71	27.26	25.71	27.26
COOMBS	67.41	67.41	51.48	53.54	56.67	56.67	56.67	56.67	56.67	56.67	56.67	56.67
RUNOFF	80.31	80.31	63.07	66.11	67.51	69.43	67.51	69.43	67.51	69.43	67.51	69.43
HARE (TIES)	85.58	85.58	71.36	85.39	83.86	87.52	83.86	87.39	83.86	87.34	83.86	87.52
COOMBS (TIES)	100.00	100.00	78.99	95.58	99.83	100.00	100.00	100.00	99.95	100.00	99.83	100.00
BALDWIN	100.00	100.00	78.86	95.62	100.00	100.00	99.83	100.00	99.78	100.00	100.00	100.00
NANSON	100.00	100.00	78.86	95.62	100.00	100.00	99.83	100.00	99.78	100.00	100.00	100.00